Fetal ECG Extraction Exploiting Joint Sparse Supports In A Dual Dictionary Framework

Furrukh Sana[§], Tarig Ballal[§], Maha Shadaydeh[‡], Ibrahim Hoteit[§], Tareq Y. Al-Naffouri[§]

[§]Division of Computer, Electrical and Mathematical Sciences and Engineering,
King Abdullah University of Science & Technology, Thuwal, Saudi Arabia.
[‡]Institute for Computer Science and Control, Hungarian Academy of Sciences, Budapest, Hungary.

***Abstract*—Electrocardiogram (ECG) signals are vital tools in assessing the health of the mother and the fetus during pregnancy. Extraction of fetal ECG (FECG) signal from the mother's abdominal recordings requires challenging signal processing tasks to eliminate the effects of the mother's ECG (MECG) signal, noise and other distortion sources. The availability of ECG data from multiple electrodes provides an opportunity to leverage the collective information in a collaborative manner. We propose a new scheme for extracting the fetal ECG signals from the abdominal ECG recordings of the mother using the multiple measurement vectors approach. The scheme proposes a dual dictionary framework that employs a learned dictionary for eliminating the MECG signals through sparse domain representation and a wavelet dictionary for the noise reduced sparse estimation of the fetal ECG signals. We also propose a novel methodology for inferring a single estimate of the fetal ECG source signal from the individual sensor estimates. Simulation results with real ECG recordings demonstrate that the proposed scheme provides a comprehensive framework for eliminating the mother's ECG component in the abdominal recordings, effectively filters out noise and distortions, and leads to more accurate recovery of the fetal ECG source signal compared to other state-of-the-art algorithms.**

***Index Terms*—Biomedical Signal Processing, Compressed Sensing, Dictionary Learning, Electrocardiogram, Fetal ECG, K-SVD, Multiple Measurement Vectors (MMV), Sparse Reconstruction, Wavelets.**

I. INTRODUCTION

Electrocardiogram (ECG) signals are vital tools in assessing the cardiac health of the mother and of the developing fetus during the course of pregnancy [1], [2], [3], [4]. While obtaining the mother's ECG (MECG) recordings using electrodes (or sensors) attached to the mother's chest is a straight forward procedure, obtaining the fetal ECG (FECG) recordings can be a much more complex procedure [5]. This is mainly because the ECG signals recorded from the chest of the mother (called thoracic ECG signals) represent the ECG signal from only the mother herself. However, the fetal ECG signal needs to be extracted from the ECG recordings measured with the electrodes placed at mother's abdomen.

Recordings from these abdominal electrodes comprise mainly of two components, the MECG and FECG, both superimposed on each other [6]. The FECG component is much smaller in magnitude compared to the MECG part. The recorded signals are also affected by distortions and noise from other sources, including those originating from the recording equipment itself. Eliminating the mother's ECG signal from these abdominal recording and extracting the fetal ECG component in the presence of noise is thus an important and challenging problem. Numerous approaches have been proposed in the literature to address this problem, such as those based on blind source separation (BSS) [5], [7], [8], [9], [10], [11], independent component analysis (ICA) [6], [12], [13], [14], [15], adaptive filtering [16], [17], sparse redundant representations [18], [19], and Wavelets [7], [20], [21], [22], [23].

In [10], a blind source separation technique is applied requiring at least three abdominal signals to separate each into MECG, FECG, and noise components. However, if the required number of abdominal ECG signals are not available or if the algorithm is unable to distinguish between the components within the abdominal recordings, the BSS approach may completely fail in separating the MECG and FECG components. In [8], another BSS technique is designed taking into account the cyclo-stationary nature of the ECG signals as useful prior information in the FECG extraction process. The application of the proposed scheme is limited by the fact that it requires prior knowledge of the cyclo-stationary frequency of the signal to be estimated.

In [15], independent component analysis (ICA) based blind source subspace separation is proposed as a tool for the extraction of the antepartum fetal electrocardiogram from multi-lead cutaneous potential recordings. As pointed out in [24], the major shortcomings of the ICA approach stem from the strong assumption on the nonstationary nature of the sources and the presence of additive noise. In [25], a system with 100 abdominal electrodes is tested where a selected subset of the electrodes is utilized for fetal ECG extraction using ICA. A specialized sensor selection algorithm is proposed that is based on a mutual information criterion. This system also inherits the aforementioned shortcomings of the ICA approach. The authors in [11] apply three different methods for BSS of maternal and fetal ECGs: principal-component analysis (PCA), higher-order singular-value decomposition (HOSVD), and higher-order eigenvalue decomposition (HOEVD). The latter two techniques belong to the ICA family and inherit the same drawbacks mentioned earlier while the former requires the sources to be uncorrelated.

In [26], the authors propose a method that employs singular value decomposition (SVD) and singular value ratio (SVR) spectrum. The elimination of the maternal ECG and determination of the fetal ECG are achieved through selective separation of the singular value decomposed components. The technique is suggested for a single ECG recording. Extension to multiple recordings is not discussed. Another major drawback of this method is that it requires the two signals to be orthogonal.

The adaptive filtering algorithm proposed in [16] uses a linear combiner (LC) to form a primary noise signal from the abdominal signals for an adaptive noise cancellation (ANC) structure. This LC assigns an adaptive weight for each signal. The weights of the ANC and LC are recursively updated such that the estimated FECG signal indicates a better waveform than the one produced by the best single abdominal signal. However, the proposed scheme does not exploit sparse domain representation for the ECG signals and utilizes a single chest sensor as the MECG reference. An adaptive filtering approach based on the wavelet transform of the ECG signals is considered in [20]. The proposed technique is, however, limited to the use of standard wavelet functions for the sparse representation of the ECG signals and does not exploit any knowledge of the structure of ECG signals. The method in [27] extends the adaptive filtering approach by including a finite impulse response (FIR) neural network in the adaptive noise cancellation scheme to provide nonlinear dynamics in the model. This method utilizes only one reference thoracic signal and one abdominal signal and does not exploit measurements from multiple electrodes.

The paper [28] considers enhancing the maternal ECG component of an abdominal ECG signal based on the continuous wavelet transform (CWT) before fetal ECG extraction. To enhance the maternal ECG component, the authors resort to optimizing the wavelet type and scale. As alluded to by the authors, the procedure may not work in a fully automated way; manual intervention might be required in some cases. In addition, the paper does not discuss actual fetal ECG extraction. Another wavelet transform based method to extract the fetal ECG signal from the composite abdominal signal is developed in [23]. Two approaches are propped: One that requires a thoracic signal, and one where no thoracic signal is needed. However, similar to [20], the proposed technique does not utilize any knowledge of the structure of ECG signals.

The authors in [18] proposed the use of multi-component dictionaries taking into consideration different structural parts of the ECG signals. However, the dictionary components are modeled based on a limited number of specific structural characteristics of the ECG signals. In [19], a dictionary learning approach for sparse redundant representation is investigated based on spatially filtering the abdominal ECG recordings while treating the fetal ECG component as noise to obtain the MECG component. The estimated MECG component is then subtracted from the abdominal recordings to extract the FECG signal. Another technique based on the well-known extended Kalman filter (EKF) is investigated in [29]. However, both the proposed techniques in [19] and [29] are based on single channel recordings and do not provide a mechanism to collectively utilize the information from multiple sensors. In [30], the authors present a literature review of interference of power line (PLI) cancellation methods. Power line interference is one of the main sources of noise that corrupts physiological signals, including the ECG, thus impairing the extraction of useful information from the signals.

The availability of ECG data from multiple electrodes provides an opportunity to leverage the information from the chest and abdominal sensors in a collaborative manner, specially when utilizing carefully selected basis for their sparse domain representation. In this work, we propose a new scheme for extracting the FECG signal from the abdominal ECG recordings of the mother by exploiting the *support*[1] similarities between the ECG recordings when they are represented in a sparse domain. This approach leads to a *Multiple Measurement Vectors* (MMV) approach [31]. The MMV approach enables more robust estimation of the MECG signals as present at the abdomen, leading to a more effective and efficient extraction of the fetal ECG signals. The proposed scheme adopts a dual dictionary framework that uses a dictionary learned using the K-SVD algorithm [32] for eliminating the MECG signals through sparse domain representation and a wavelet dictionary for the noise reduced sparse estimation of the fetal ECG signals. The K-SVD algorithm is a generalization of the K-means clustering method and uses the SVD (Singular Value Decomposition) to find a dictionary of atoms for sparse representation of training signals [32]. We also present a novel methodology of combining the FECG signal estimates from the individual sensors, again leveraging the joint support feature, to infer a single estimate of the FECG signal. Results obtained using real ECG recordings demonstrate that the proposed MMV-based dual dictionary approach provides a comprehensive framework for eliminating the MECG components in the abdominal recordings, effectively filters out noise and distortions, and leads to accurate recovery of the fetal ECG signal.

II. THE MULTIPLE MEASUREMENT VECTORS (MMV) FRAMEWORK

The MMV [31] is a sparse signal estimation framework in which signals exhibiting a common support $\mathcal{S}$ (i.e. locations of the non-zero values) are jointly estimated using measurements available from multiple sensors. The MMV signal model is given as

$$\mathbf{Y} = \mathbf{\Psi}\mathbf{X} + \mathbf{W}, \tag{1}$$

where $\mathbf{X} \in \mathbb{C}^{L \times N}$ is a collection of N sparse vectors of length L to be estimated from equal number of measurements $\mathbf{Y} \in \mathbb{C}^{M \times N}$ of length M based on the sensing matrix $\mathbf{\Psi} \in \mathbb{C}^{M \times L}$. The matrix $\mathbf{W} \in \mathbb{C}^{M \times N}$ represents the noise that contaminates the observed signals. The MMV formulation makes use of the fact that the non-zero values in any of the vectors in $\mathbf{X}$ occur at the same locations, as illustrated in Figure 1. The joint estimation framework of the MMV based algorithms helps improve the quality of the estimated signals using collaborative estimation formulations.

The MMV approach is well suited for the FECG extraction problem since there are multiple observations of the ECG data available from the different electrodes placed at the abdomen. Moreover, in a sparse domain represented by an over-complete dictionary of basis elements, ECG signals can

[1]The *support* $\mathcal{S}$ of a signal is defined as the locations of the non-zero values within the signal.

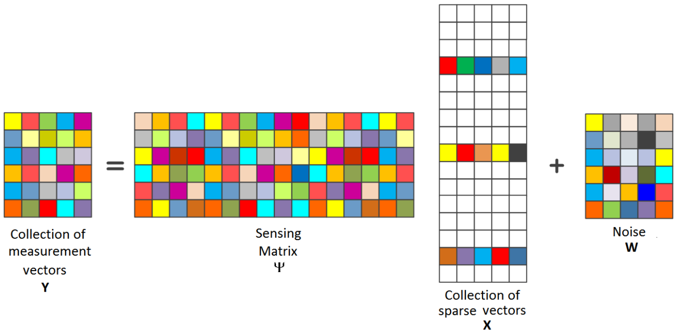

Fig. 1. Illustration of the MMV based sparse signal estimation problem. The colored boxes represent non-zero values.

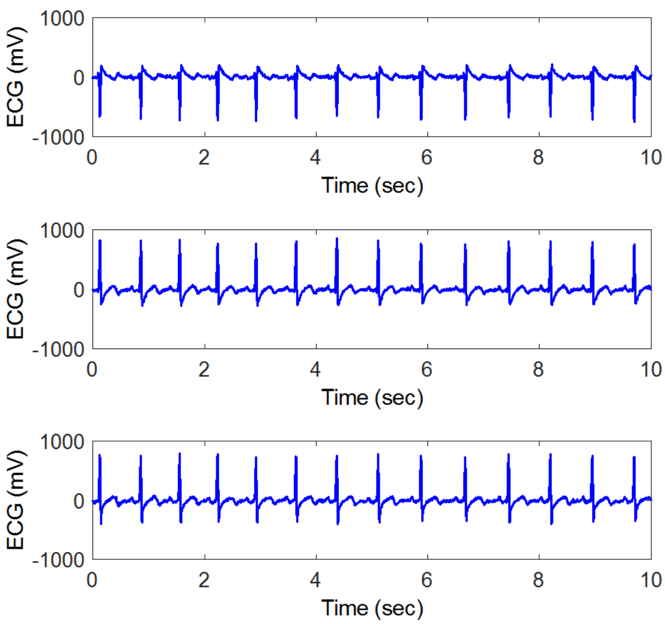

Fig. 2. Top to bottom: ECG signals recorded from the electrodes 1, 2, and 3, respectively placed at the chest of a pregnant woman.

be represented using a sparse linear combination of the basis elements. Consider the ECG recordings obtained from the chest and abdomen of a pregnant woman (real dataset available online [33]) as shown in Figure 2 and Figure 3, respectively. These ECG recordings can be represented in a sparse domain, for example, of Daubechies wavelets[2] basis function with as low as 2% sparsity rate and reconstructed with significant accuracy as shown in Figure 4. The ECG signals obtained from the different electrodes share a significant percentage of the support $\mathcal{S}$, i.e. the signals are mostly made up of the same basis elements. For the abdominal ECG signals shown in Figure 3, we found (through Matlab) that they share up to 80% of the basis elements when represented in the Daubechies wavelets domain. This similarity in support is also evident by looking at the plot of coefficients in Figure 5. In this work, we exploit this property of their common support in joint estimation of the ECG signals using a MMV based approach.

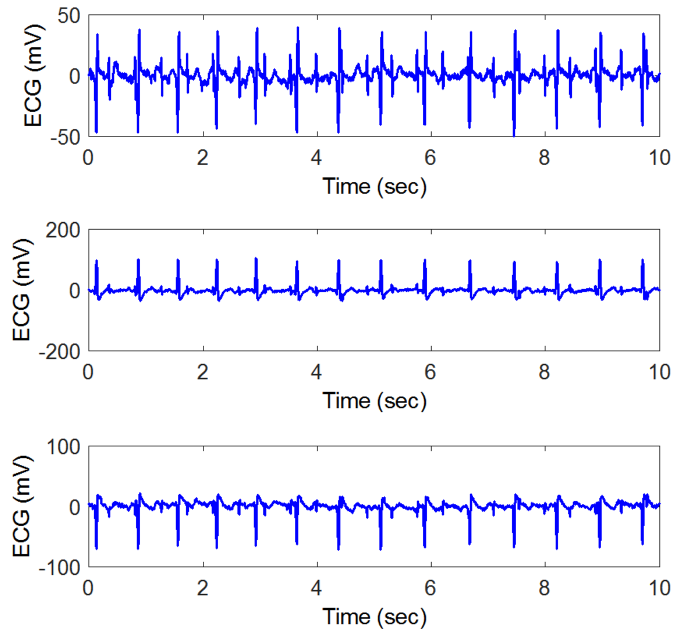

Fig. 3. Top to bottom: ECG signals recorded from the electrodes 1, 2, and 3, respectively placed at the abdomen of a pregnant woman.

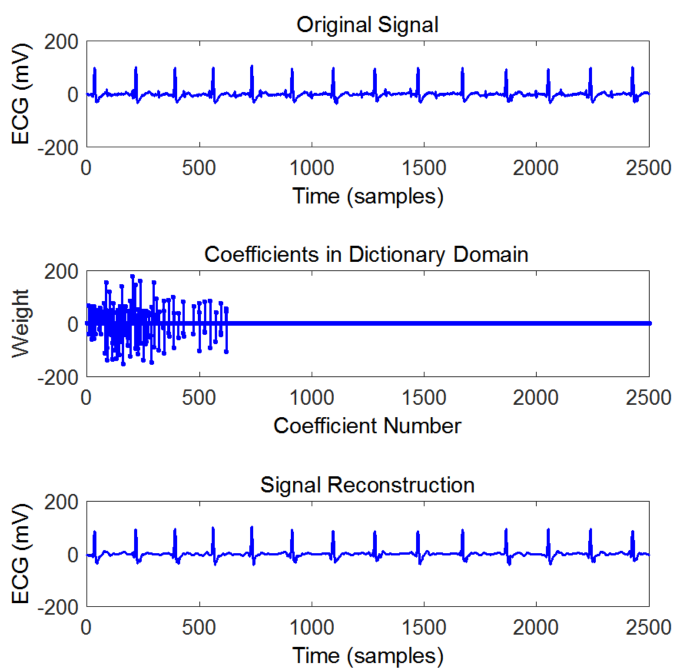

Fig. 4. An example of reconstructed abdominal ECG signal using the Daubechies wavelets. Top: Observed ECG signal in time-domain, Middle: ECG signal represented in wavelet domain with a 2% sparsity rate, Bottom: Reconstructed signal from the wavelet coefficients in time-domain.

The multiple measurements from the chest and abdominal sensors can be used to estimate the MECG component in the abdominal ECG recordings using an MMV algorithm while treating the FECG component as noise. Once estimated, MECG component can then be subtracted from the abdominal recordings, leading to a more robust estimation of the fetal ECG signal. Furthermore, the same joint support characteris-

[2]Daubechies wavelets are one of the well known and commonly used wavelet functions. For our demonstration in Figure 4, we used the db4 wavelet functions which are Daubechies wavelets with 4 vanishing points.

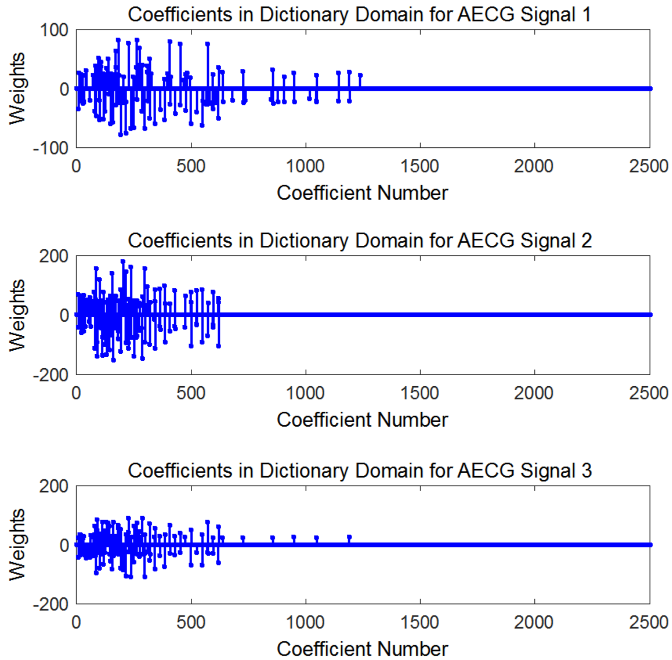

Fig. 5. Top to Bottom: Wavelet coefficients for the abdominal recordings from sensor 1, 2, and 3, respectively shown in Figure 3.

tics of the FECG signal estimates from the different abdominal sensors can be used to infer a single and more accurate estimate of the FECG source signal, as discussed in the next section.

III. JOINT ESTIMATION OF THE FETAL ECG SIGNAL

A. Problem Formulation

We begin by first representing the MECG signal '$\mathbf{m}$' in a sparse domain using an over-complete dictionary $\mathbf{\Psi}_M$. Mathematically this is given by the expression,

$$\mathbf{m} = \mathbf{\Psi}_M \mathbf{d}_M \tag{2}$$

where $\mathbf{d}_M$ is a sparse vector of coefficients used for representing the MECG signal in the sparse domain given by the basis elements of the matrix $\mathbf{\Psi}_M$.The MECG signal can be recorded directly using the sensors placed at the chest of the mother giving the measurements

$$\mathbf{y}_M^j = \mathbf{m} + \mathbf{w}^j, \tag{3}$$

or

$$\mathbf{y}_M^j = \mathbf{\Psi}_M \mathbf{d}_M + \mathbf{w}^j \tag{4}$$

where $\mathbf{w}^j$ is the additive white Gaussian noise in the measurement recorded from the j^{th} chest sensor.

The ECG recordings at the abdominal sensors on the other hand are a combination of the MECG signal $\mathbf{m}$ superimposed with the fetal ECG signal $\mathbf{f}$ and noise. Mathematically this is given by the expression

$$\mathbf{y}_A^i = \mathbf{m} + \mathbf{f} + \mathbf{n}^i, \tag{5}$$

or,

$$\mathbf{y}_A^i = \mathbf{\Psi}_M \mathbf{d}_M + \mathbf{f} + \mathbf{n}^i, \tag{6}$$

where $\mathbf{y}_A^i$ is the ECG recording obtained from the i^{th} abdominal sensor for $i = 1, 2, .., N$ and $\mathbf{n}^i$ is additive white Gaussian noise in the measurement from the corresponding sensor. The attenuation of the MECG signal as it travels from the chest of the mother towards the abdomen is assumed negligible and hence ignored in the above formulation.

As such, the problem of extracting the FECG signal from the abdominal recordings is converted to estimating the MECG signal component and eliminating it, along with the noise and distortions, to recover the FECG part.

B. The Fetal ECG Extraction Algorithm

We utilize the availability of multiple observations from the different abdominal sensors which can be aggregated in a single matrix as

$$\mathbf{Y}_A = [\mathbf{y}_A^1, \mathbf{y}_A^2, .., \mathbf{y}_A^N] \tag{7}$$

to employ the MMV framework in estimating the MECG signal. Since direct ECG recordings of the mother are available from the thoracic electrodes, we can use them to learn a suitable dictionary $\mathbf{\Psi}_M$ for efficient and accurate representation of the MECG signals in a sparse domain. For this purpose, we use the K-SVD dictionary learning algorithm [32] with the MECG recordings $\mathbf{y}_M$ available from the mother's *chest* as the training set. After constructing the dictionary, the goal is to project the measurements $\mathbf{y}_A$ available from the *abdomen* onto the dictionary to obtain the sparse representation of the MECG component in these measurements. The estimation problem in this sparse domain is defined as estimating the set of vectors $\hat{\mathbf{D}}_M = [\hat{\mathbf{d}}_M^1, \hat{\mathbf{d}}_M^2, .., \hat{\mathbf{d}}_M^N]$ that corresponds to the set of abdominal measurements $\mathbf{Y}_A$. The FECG signal is considered as noise for the purpose of this step.

$$\hat{\mathbf{D}}_M = \mathbf{MMV}(\mathbf{Y}_A, \mathbf{\Psi}_M). \tag{8}$$

The set of coefficient vectors $\hat{\mathbf{D}}_M$ can be estimated using a MMV algorithm. We utilize the *'MMV-Support Agnostic Bayesian Matching Pursuit'* (M-SABMP) algorithm [31] for this purpose. The M-SABMP is a sparse signal recovery algorithm based on the multiple measurement vector approach and allows to estimate a sparse signal by jointly exploiting observations from multiple sensors. Once the coefficients are estimated, they can be used to reconstruct the time-domain MECG signal as present at the abdomen

$$\hat{\mathbf{M}} = \mathbf{\Psi}_M \hat{\mathbf{D}}_M \tag{9}$$

to subsequently eliminate from the abdominal ECG recordings, yielding the *initial* estimates of the FECG signals

$$\tilde{\mathbf{F}} = \mathbf{Y}_A - \hat{\mathbf{M}}, \tag{10}$$

where $\tilde{\mathbf{M}} = [\tilde{\mathbf{m}}^1, \tilde{\mathbf{m}}^2, .., \tilde{\mathbf{m}}^N]$ and $\tilde{\mathbf{F}} = [\tilde{\mathbf{f}}^1, \tilde{\mathbf{f}}^2, .., \tilde{\mathbf{f}}^N]$ represent the mother and fetal ECG signal estimates from all abdominal sensors, respectively.

However, even after the removal of the MECG component, the estimated FECG signals, here denoted by $\tilde{\mathbf{F}}$, may still

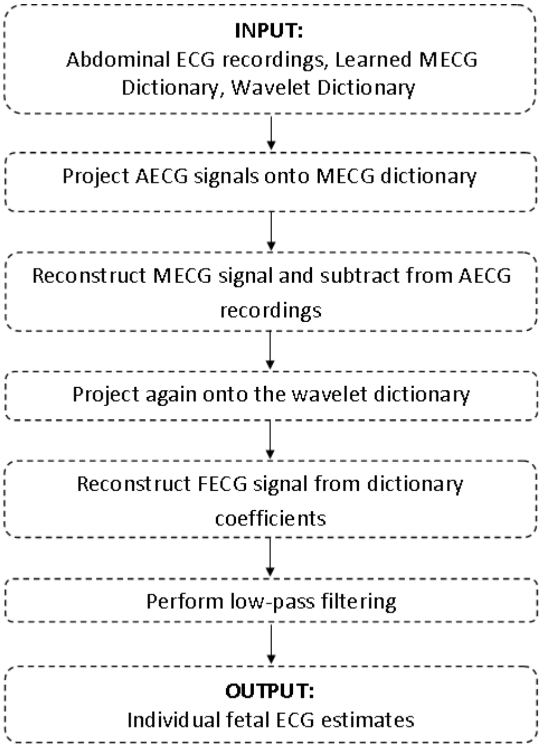

Fig. 6. Flowchart representation of Algorithm 1.

Algorithm 1 Fetal ECG Extraction Algorithm

- Input: Abdominal ECG recordings $\mathbf{Y}_A$, MECG dictionary $\mathbf{\Psi}_M$ learned using the K-SVD algorithm, Wavelet dictionary $\mathbf{\Psi}_{Wavelet}$
- **Step 1:** Project abdominal recordings onto the MECG dictionary:

$$\hat{\mathbf{D}}_M = \mathbf{MMV}(\mathbf{Y}_A, \mathbf{\Psi}_M).$$

- Reconstruct MECG signal and subtract from original abdominal recordings:

$$\tilde{\mathbf{F}} = \mathbf{Y}_A - \hat{\mathbf{M}},$$

where $\hat{\mathbf{M}} = \mathbf{\Psi}_M \hat{\mathbf{D}}_M$.
- **Step 2:** Project again onto the wavelet dictionary:

$$\hat{\mathbf{D}}_{\tilde{F}} = \mathbf{MMV}(\tilde{\mathbf{F}}, \mathbf{\Psi}_{Wavelet}).$$

- Reconstruct distortion-minimized FECG signal estimates:

$$\hat{\mathbf{F}}_{noisy} = \mathbf{\Psi}_{Wavelet} \hat{\mathbf{D}}_{\tilde{F}}.$$

- **Step 3:** Perform low-pass filtering:

$$\hat{\mathbf{F}} = LPF(\hat{\mathbf{F}}_{noisy}).$$

- Output: $\hat{\mathbf{F}} := [\hat{\mathbf{f}}^1, \hat{\mathbf{f}}^2, .., \hat{\mathbf{f}}^N]$.

contain high levels of noise and distortion. To eliminate these effects, we project the estimates from the last step once more, but this time onto a dictionary of wavelet basis elements $\mathbf{\Psi}_{Wavelet}$. Wavelets basis have been useful for signal recovery in a variety of applications, including ECG feature extraction and de-noising [14], [34]. Different wavelet functions help preserve the shape and characteristic features of a signal in a particular application by representing the signal in a domain defined by wavelet basis functions most suitable for that particular signal, thereby eliminating the features corresponding to noise and distortions. By carefully selecting the wavelet basis functions such that they capture the significant features of the ECG signal, we can ensure that only the signal values representing the fetal ECG signals are preserved in the sparse domain while the effects of distortion are minimized. The coefficients $\hat{\mathbf{D}}_{\tilde{\mathbf{F}}}$ selected for sparse representation of $\tilde{\mathbf{F}}$ in the wavelet domain are used to generate a distortion free version of the fetal ECG signals

$$\hat{\mathbf{F}}_{noisy} = \mathbf{\Psi}_{Wavelet} \hat{\mathbf{D}}_{\tilde{\mathbf{F}}}, \tag{11}$$

which are passed through a low-pass filter with a cut-off frequency of 5000 Hz to give the estimates of the fetal ECG signals from the individual sensors as

$$\hat{\mathbf{F}} = LPF(\hat{\mathbf{F}}_{noisy}), \tag{12}$$

where $\hat{\mathbf{F}} = [\hat{\mathbf{f}}^1, \hat{\mathbf{f}}^2, .., \hat{\mathbf{f}}^N]$ represent the final FECG signal estimates from the individual abdominal sensors. The complete FECG extraction procedure is summarized in Algorithm 1 and depicted in Figure 6.

C. The Fetal ECG Combination Algorithm

The above procedure allows to obtain the FECG signal estimates from the individual abdominal sensors. As these individual signals are the estimates of a single FECG source, it is desirable to combine them in an appropriate manner to recover the FECG source signal. These estimates from different sensors might offer different estimation accuracies based on the quality of the individual measurements. It is important to take this into consideration when combining these contributions to infer the FECG source signal. We identify two important considerations when combining the individual sensor estimates: (i) the support of the individual sensor estimates should be common, and (ii) the amplitudes for the support elements should be combined using a weighted linear combination based on the quality of the individual estimates.

For this purpose, we first perform a scaling and alignment operation on the estimated FECG and MECG (as estimated at the abdomen) signals such that their combination regenerates close replicas of the observed abdominal ECG (AECG) signal. To this end, we define the residual error for each i^{th} abdominal sensor as

$$R^i = \| \hat{\mathbf{y}}_A^i - \mathbf{y}_A^i \|_2^2 \tag{13}$$

where,

$$\hat{\mathbf{y}}_A^i = \alpha.\text{Shift}\left(\hat{\mathbf{y}}_M^i, \tau_1\right) + \beta.\text{Shift}\left(\hat{\mathbf{f}}^i, \tau_2\right), \tag{14}$$

α, β are the scaling coefficients and τ_1, τ_2 specify the amount by which the estimated MECG and FECG signals are shifted using the operator Shift(.), respectively. We then solve the

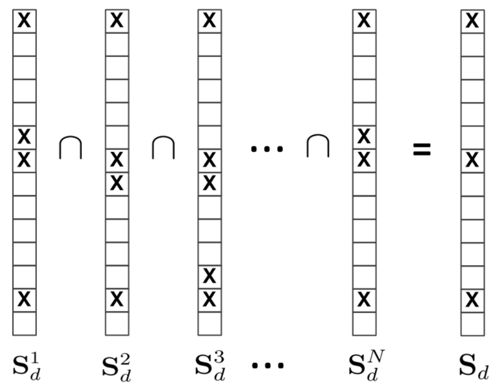

Fig. 7. Illustration of determining the common support from the support of the individual sensor estimates. The crossed boxes indicate the *locations* of the non-zero values.

minimization problem

$$\min_{\alpha,\beta,\tau_1,\tau_2} \parallel R^i \parallel_2^2 \tag{15}$$

or, in a complete form,

$$\min_{\alpha,\beta,\tau_1,\tau_2} \parallel \alpha.\text{Shift}\left(\hat{\mathbf{y}}_M^i,\tau_1\right)+\beta.\text{Shift}\left(\hat{\mathbf{f}}^i,\tau_2\right)-\mathbf{y}_A^i \parallel_2^2 \tag{16}$$

for each i^{th} sensor. The above equation can be solved by constructing a search space over a limited range of values for each of the variables α,β,τ_1, and τ_2 to find the combination yielding the least residual error. Although there are no theoretical limits to the number of values that can be considered for shifting and alignment process, we found that optimizing over 2-3 samples to both left and right shift directions and between 1.0 to 1.5 scaling factors proved sufficient for alignment purposes.

The residual error helps determine the quality of the signals estimated from each sensor. The residual error is inversely related to the reconstruction quality and hence can be used to determine the weights for the linear combination as

$$\dot{W}^i = \frac{1}{R^i}. \tag{17}$$

However, for the sum of weights to equal unity, each weight is normalized as

$$W^i = \frac{1/R^i}{1/R^1+1/R^2+..+1/R^N}. \tag{18}$$

Next, we determine the common support of the estimated FECG signals. Ideally, this task would be performed perfectly by the MMV estimation algorithm itself. However, practical limitations, such as unsuitable positioning of the measurement sensors, can result in small non-zero values at different locations of the estimated vector. To achieve absolute support similarity, we project all sensors estimates on a domain represented by a simple identity matrix $\mathbf{\Psi_I}$ and retain values corresponding only to the support locations that are common to all vector of coefficients $\mathbf{d}^1,\mathbf{d}^2,..,\mathbf{d}^N$, while setting the rest to zero. The process is illustrated in Figure 7 and mathematically given as,

$$\mathcal{S}_d = \mathcal{S}_d^1 \cap \mathcal{S}_d^2 \cap ...\mathcal{S}_d^N. \tag{19}$$

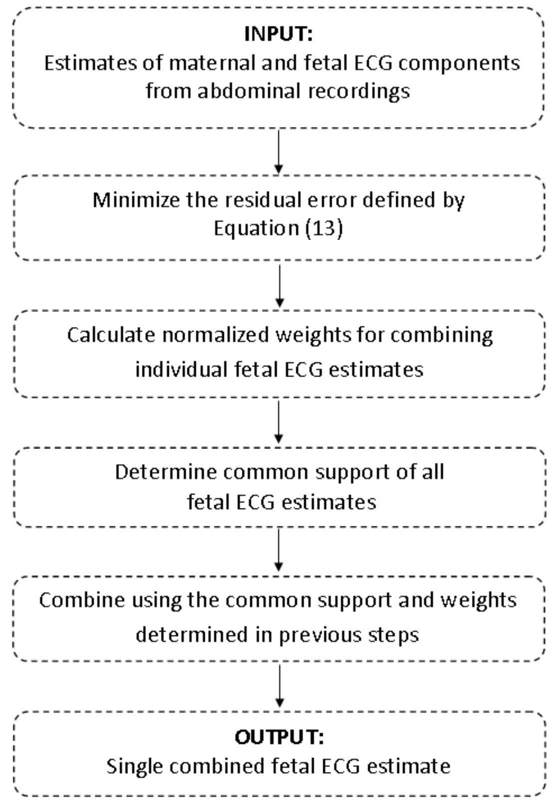

Fig. 8. Flowchart representation of Algorithm 2.

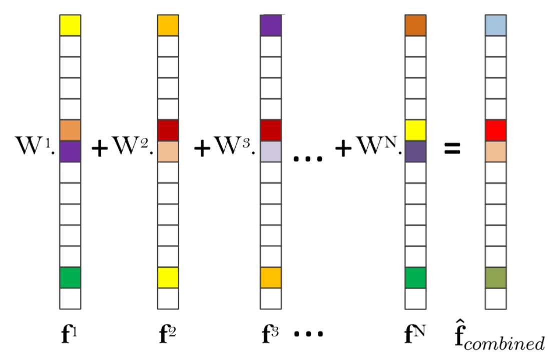

Fig. 9. Illustration of the weighted linear combination of the individual FECG signal estimates.

The single source FECG signal is then simply reconstructed in the time domain, as illustrated in Figure 9, using

$$\hat{\mathbf{f}}_{combined} = \mathbf{\Psi_I} * \left(W^1 d^1_{\mathcal{S}_d} + W^2 d^2_{\mathcal{S}_d} + ... + W^N d^N_{\mathcal{S}_d}\right), \tag{20}$$

where the coefficients $d^i_{\mathcal{S}_d}$ for the i^{th} signal have non-zero values only at the support locations given by $\mathcal{S}_d$. The complete steps for combining the sensor estimates are summarized in Algorithm 2 and depicted in Figure 8.

.

IV. SIMULATION RESULTS

We validate the performance of our proposed algorithm using two real datasets available online that represent cutaneous potential recordings of two pregnant women. We note that our proposed technique is not sensitive to any pre-described

Algorithm 2 Algorithm for Combining Individual FECG Signal Estimates

- Define the residual error:

$$R^i = \| \hat{\mathbf{y}}_A^i - \mathbf{y}_A^i \|_2^2,$$

where

$$\hat{\mathbf{y}}_A^i = \alpha.\text{Shift}\left(\hat{\mathbf{y}}_M^i, \tau_1\right) + \beta.\text{Shift}\left(\hat{\mathbf{f}}^i, \tau_2\right).$$

- Minimize the residual error for each individual sensor estimate:

$$\min_{\alpha,\beta,\tau_1,\tau_2} \| \alpha.\text{Shift}\left(\hat{\mathbf{y}}_M^i, \tau_1\right) + \beta.\text{Shift}\left(\hat{\mathbf{f}}^i, \tau_2\right) - \mathbf{y}_A^i \|_2^2 .$$

- Calculate the normalized weights for linear combination:

$$W^i = \frac{1/R^i}{1/R^1 + 1/R^2 + .. + 1/R^N}.$$

- Determine common support for all sensor estimates when represented in domain $\mathbf{\Psi_I}$:

$$\mathcal{S}_d = \mathcal{S}_d^1 \cap \mathcal{S}_d^2 \cap \mathcal{S}_d^3 \cap ... \mathcal{S}_d^N .$$

- Reconstruct combined FECG source signal:

$$\hat{\mathbf{f}}_{combined} = \mathbf{\Psi_I} * \left(W^1 d^1_{\mathcal{S}_d} + W^2 d^2_{\mathcal{S}_d} + ... + W^N d^N_{\mathcal{S}_d}\right).$$

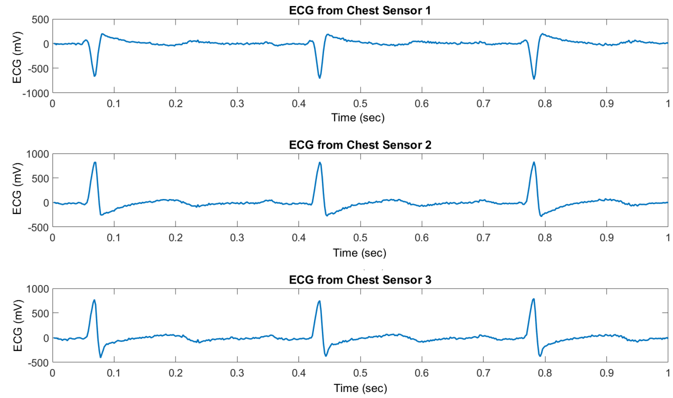

Fig. 10. Top to Bottom: MECG signals from Real Dataset 1 recorded using chest sensors 1, 2, and 3, respectively.

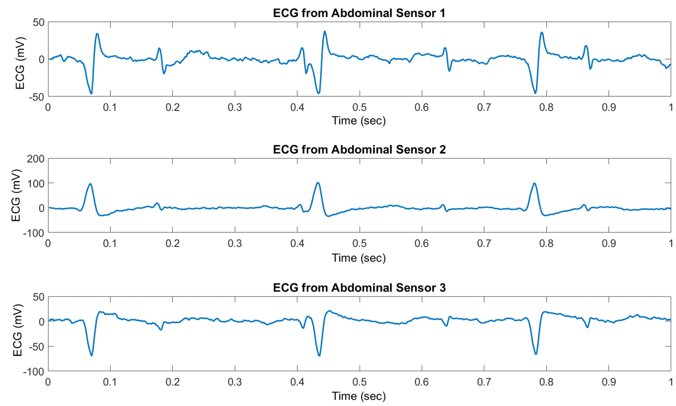

Fig. 11. Top to Bottom: AECG signals from Real Dataset 1 recorded using abdominal sensors 1, 2, and 3, respectively.

positioning of the electrodes. As long as electrodes at the chest can detect and record maternal ECG signal and electrodes at abdomen can detect and record the composite maternal plus fetal ECG signal, our algorithm can extract the fetal ECG signal. For quantitative analysis, we assess the performances of our proposed technique in terms of *True Positives* (TP := number of fetal ECG pulses estimated at correct locations), *False Positives* (FP := number of fetal ECG pulses detected at locations where they should not be), and *False Negatives* (FN := number of fetal ECG pulses missed from estimation at the correct location) estimation events. Marking these events in the recovered fetal ECG signal helps in further computing the statistical measures of *Precision*, *Recall*, and *F1-score* which are computed as

$$\text{Precision} = \frac{TP}{TP + FP}, \tag{21}$$

$$\text{Recall} = \frac{TP}{TP + FN}, \tag{22}$$

$$\text{F1-score} = 2.\frac{\text{Precision.Recall}}{\text{Precision} + \text{Recall}}. \tag{23}$$

To compare the proposed technique with other methods, we also applied the BSS technique presented in [10], adaptive filtering technique presented in [16], and the K-SVD based denoising technique presented in [19] to the same datasets. These techniques and their limitations were discussed briefly in Section I.

1) Real Dataset 1: The first dataset comprises recordings from 8 channels, each of 5 seconds duration sampled at a rate of 500 Hz [33]. Three ECG signals correspond to the maternal ECG (MECG) signals recorded using electrodes placed at the chest of the pregnant woman. These are partially shown in Figure 10. These MECG signals are divided into smaller signals using windows of 0.25 second durations and used as the training set to generate the dictionary $\mathbf{\Psi}_M$ for representing the mother's ECG signals in the sparse domain. This choice of window size was made so that a sufficient number of training signals are available from the limited 5 seconds of ECG recordings. The total number of resulting training signals from the three electrodes is 60, which were then used to generate the dictionary of 30 linearly independent basis elements using the K-SVD algorithm [32]. We note that it is important to use a small error tolerance value while learning the dictionary using the K-SVD algorithm.

The remaining five cutaneous ECG recordings are available from the electrodes placed at the abdomen of the pregnant woman and correspond to the ECG signals which are the combination of the maternal ECG signal superimposed on the fetal ECG signal along with noise and other interference sources. We use the first three recordings from these abdominal signals, which are partially shown in Figure 11.

We apply our proposed fetal ECG extraction technique of

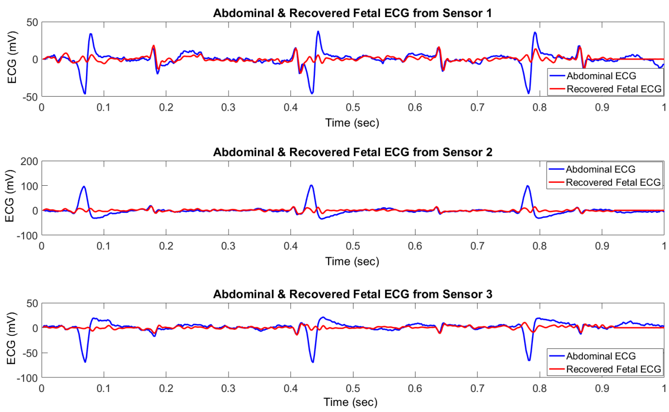

Fig. 12. Top to Bottom: Initial fetal ECG estimates from sensor 1, 2, and 3, respectively after the 1st stage of the proposed extraction scheme in Algorithm 1 for Real Dataset 1 (Blue: Abdominal ECG, Red: Recovered fetal ECG).

Algorithm 1 by first projecting the abdominal ECG recordings on the dictionary $\mathbf{\Psi}_M$ using the M-SABMP algorithm [31] and subtract the reconstructed MECG signals from the original abdominal recordings. A sparsity rate of 25% is used with the M-SABMP algorithm for this step. Although this single step is able to significantly eliminate the MECG component in the abdominal recordings, the recovered fetal ECG still suffers from large noise and distortions as shown in Figure 12.

Proceeding to the second stage of the proposed technique in Algorithm 1, the recovered fetal ECG signals are now further projected onto a wavelet dictionary to mitigate the effects of noise and any residual of MECG pulses left in the results from the first step. We use the simple Kronecker delta function as the wavelet of choice as it allows to process every sample data point individually. By using a tight sparsity constraint with the MMV based estimation algorithm, we force the algorithm to retain only a few most significant values within each window. This helps in preserving the signal values representing the fetal ECG signals while effectively eliminating the low amplitude distortions (such as those from noise or residual of a suppressed MECG pulse) using a 4% sparsity rate with the wavelet dictionary. A final post-processing of the recovered estimates using a low-pass filter provides the extracted FECG signals from the individual sensors shown in Figure 13.

The technique proposed in Algorithm 2 is applied for combining these individual estimates, calibrating the scales and alignment such that the residuals between the observed and reconstructed abdominal signals are minimized. The common support is determined once again using the M-SABMP algorithm and the weights obtained from residual calculation are used to finally combine the individual sensor estimates to provide the final estimate of the FECG source signal as shown in Figure 14. As clearly demonstrated, the proposed algorithms help significantly in eliminating the MECG component, noise, and distortions, providing a very clean and accurate estimate of the FECG source signal.

The proposed technique is compared with the BSS, adaptive filtering, and the K-SVD based denoising techniques applied to the same dataset. The BSS and the denoising techniques provide three fetal ECG estimates from the three abdominal

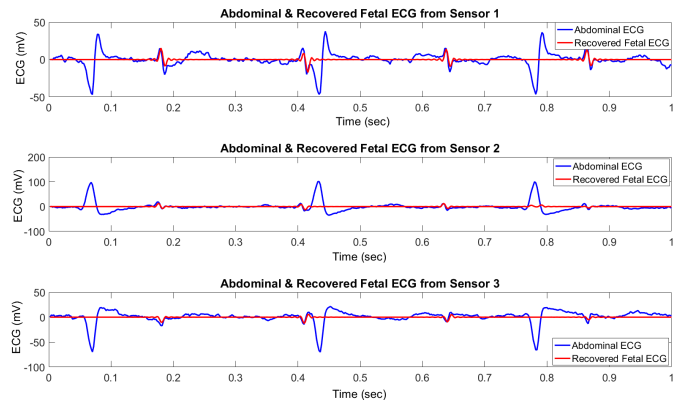

Fig. 13. Top to Bottom: Individual fetal ECG signal estimates from sensor 1, 2, and 3, respectively recovered after complete steps of the proposed extraction scheme in Algorithm 1 for Real Dataset 1 (Blue: Abdominal ECG, Red: Recovered fetal ECG).

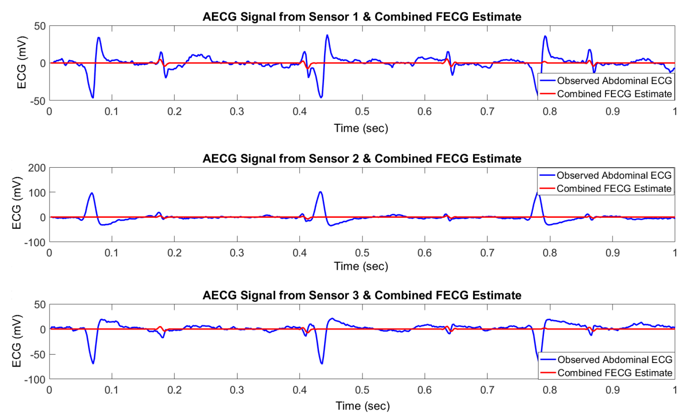

Fig. 14. Top to Bottom: Combined estimate of the fetal ECG source signal plotted over the abdominal signal from sensor 1, 2, and 3, respectively for Real Dataset 1 (Blue: Abdominal ECG, Red: Combined estimate of the fetal ECG).

recordings used from this dataset. However, due to limited space, the best estimated fetal ECG signal from each of these techniques is plotted in Figure 15 along with the single FECG signal estimated using the adaptive filtering technique and the combined fetal ECG estimate obtained using the proposed technique of this paper. Note that the amplitudes of the fetal ECG signals estimated using the four techniques differ significantly from each other and from amplitude levels in the abdominal recordings. To facilitate visual inspection and analysis (for qualitative purposes only), these amplitudes were scaled so that the FECG pulses in the estimated signals and in the abdominal ECG recordings appear at similar levels.

While estimation qualities from these techniques appear to be comparable to the proposed technique, the algorithms and their performances suffer from some limitations. The BSS technique works by separating abdominal signals into MECG, FECG, and noise components. Once these signal components are separated, the FECG component needs to be manually identified before further processing can take place. The estimates from the denoising approach are only partially accurate as the algorithm fails in recovering half of the fetal

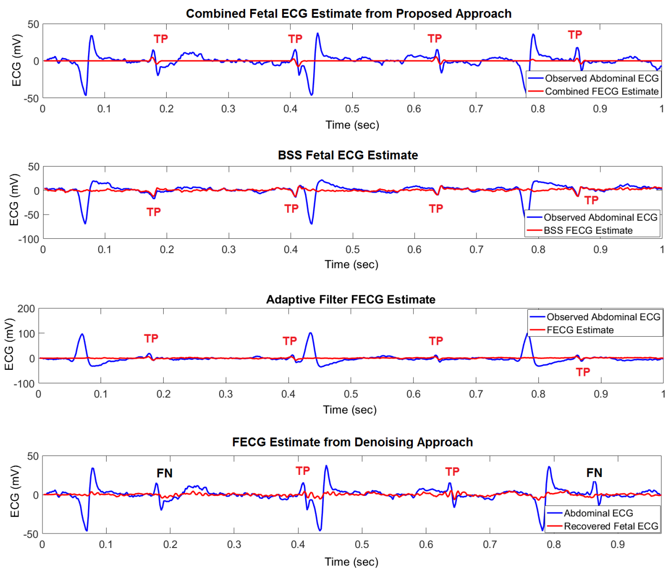

Fig. 15. Top to Bottom: Performance comparison of the proposed technique with the BSS, Adaptive filtering, and Denoising approaches, respectively using Real Dataset 1 (Blue: Abdominal ECG, Red: Estimated fetal ECG). TP marks a 'True Positive' event, FP a 'False Positive' event, while FN marks a 'False Negative' event.

TABLE I
PERFORMANCE COMPARISON OF THE PROPOSED APPROACH WITH BSS, ADAPTIVE FILTERING, AND DENOISING APPROACHES USING REAL DATASET 1 IN TERMS OF STATISTICAL MEASURES OF TP, FP, FN, PRECISION, RECALL, AND F1-SCORE.

	TP	FP	FN	Precision	Recall	F1
Proposed Approach	4	0	0	1	1	1
BSS Approach	4	0	0	1	1	1
Adaptive Filtering	4	0	0	1	1	1
Denoising Approach	2	0	2	1	0.5	0.67

ECG pulses. The FECG signal estimated using the adaptive filtering technique performs the closest to the proposed MMV based approach, although still demonstrating some levels of noise.

Figure 15 also allows to mark the recovered fetal ECG signal in terms of *'True Positives'*, *'False Positives'*, and *'False Negatives'* estimation events. These events are marked in Figure 15 with respective abbreviations[3]. The values for statistical measures of *Precision*, *Recall*, and *F1-score* are then computed and compared with the values for fetal ECG signals recovered from all other algorithms.

The four techniques are compared in terms of these measures in the table shown in Table I which shows a perfect F1 score for all but the denoising based estimation approach. Comparison using this dataset suggests that the proposed technique performs at least as good as the state-of-the-art BSS and adaptive filtering algorithms.

2) Real Dataset 2:

[3]The development of an automated algorithm for counting True Positive, False Positive, False Negative events is outside the scope of this manuscript and such scoring has been done manually by visually inspecting if the FECG pulses in the estimated signals overlap FECG pulses in the abdominal signals or not.

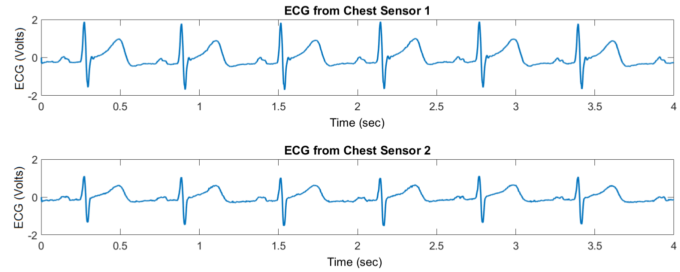

Fig. 16. Top to Bottom: MECG signals from Real Dataset 2 recorded using chest sensors 1 and 2 respectively.

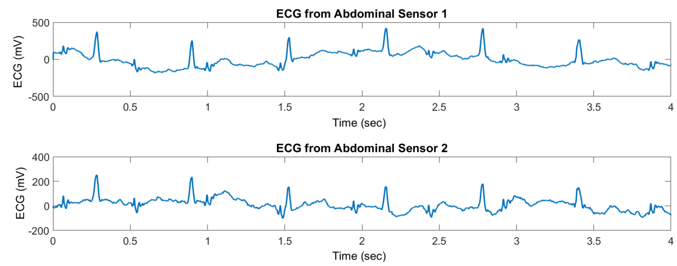

Fig. 17. Top to Bottom: AECG signals from Real Dataset 2 recorded using abdominal sensors 1 and 2 respectively.

For our second dataset of real ECG recordings, we use the 'ecgca906.edf' ECG data available from the PhysioNet online database [35]. This dataset provides about 33 minutes of ECG recordings sampled at a rate of 1 KHz. Only two MECG signals recorded using electrodes placed at the chest are available in this dataset and these are partially shown in Figure 16. As before, the MECG signals are divided using windows of 0.25 second durations and used as the training set to generate the dictionary $\boldsymbol{\Psi}_M$ for representing the mother's ECG signals. Other window sizes did not appear to significantly affect the estimation performance of the proposed technique. These settings resulted in a total number of 720 training signals when utilizing 90 seconds of ECG recordings for a given simulation, which were then used to generate a dictionary of 360 linearly independent basis elements using the K-SVD algorithm [32].

Four cutaneous ECG recordings are available from the electrodes placed at the abdomen of the pregnant woman. However, to demonstrate that our multiple measurement vectors based approach can work with data from as low as two electrodes, we only utilize two of these abdominal recordings which are partially shown in Figure 17. As before, we first apply our proposed fetal ECG extraction technique of Algorithm 1. The individual fetal ECG signals estimated using the real dataset 2 are shown in Figure 18. We then proceed with the combination of these individual fetal ECG estimation using Algorithm 2 with the final combined fetal ECG estimate shown in Figure 19.

Once again, we compare the proposed technique with the BSS, adaptive filtering, and denoising approaches in Figure 20 using the real dataset 2. As before, only the best fetal ECG estimate from the BSS and denoising techniques is plotted along with the estimated FECG signal from the adaptive filtering algorithm and the combined fetal ECG estimate

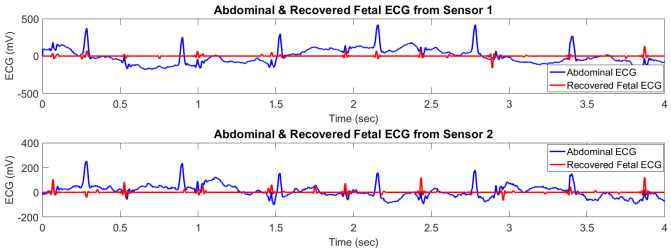

Fig. 18. Top to Bottom: Individual fetal ECG signal estimates from abdominal sensor 1 and 2 respectively recovered after complete steps of the proposed extraction scheme in Algorithm 1 for Real Dataset 2 (Blue: Abdominal ECG, Red: Recovered fetal ECG).

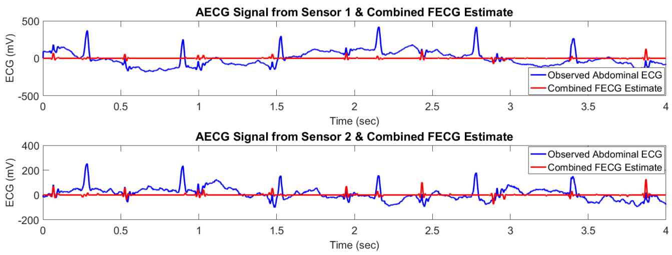

Fig. 19. Top to Bottom: Combined estimate of the fetal ECG source signal plotted over the abdominal signal from sensor 1 and 2 respectively for Real Dataset 2 (Blue: Abdominal ECG, Red: Combined estimate of the fetal ECG).

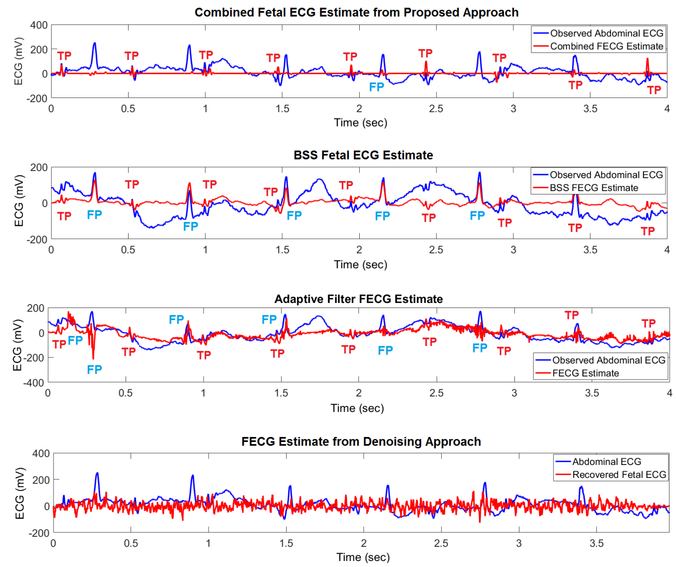

Fig. 20. Top to Bottom: Performance comparison of the proposed technique with the BSS, Adaptive filtering, and Denoising approaches, respectively using real dataset 2 (Blue: Abdominal ECG, Red: Estimated fetal ECG). TP marks a 'True Positive' event, FP a 'False Positive' event, while FN marks a 'False Negative' event. Note that a TP event is marked around 3.5 seconds because of detection of a FECG pulse that was overlapped by a MECG pulse in the abdominal recording

obtained using the proposed technique. The dataset from PhysioNet proves to be far more challenging for all tested algorithms. The BSS technique was implemented using three abdominal signals (compared to two for all other techniques) since it requires at least three abdominal signals to separate each into MECG, FECG, and noise components. However the BSS algorithm, which is often considered state-of-the-art, still failed to separate the MECG and FECG signals, giving estimated signals which are just the abdominal ECG signals with less noise. The estimation performance from the adaptive filter is not much different either. Although it preserves the fetal ECG pulses at the right location, it fails to suppress the MECG pulses at most of the locations. The denoising approach also fails completely as the estimated signal is too random to derive any useful information. The proposed technique performs better than all other algorithms with real dataset 2 as well, successfully detecting all FECG pulses at the correct locations and eliminating MECG pulses from the abdominal ECG recordings with only one false positive event.

Similar observations and conclusion can be made when analyzing the performances of the algorithms in terms of the statistical measures of Precision, Recall, and F1 values. These values are presented in the table shown in Table II for real dataset 2. It is to be noted that there is a TP event marked around 3.5 seconds in Figure 20 because of detection of a FECG pulse that was overlapped by a MECG pulse in the abdominal recording. While the BSS and adaptive filtering algorithms are able to achieve the perfect TP score of 9 for the length of the signal shown in Figure 20, they also result in high rates of false positives. The signal estimated using the denoising approach was not scored due to lack of usefulness. The proposed MMV based approach outperforms all the other algorithms, both in terms of Precision and F1 scores, while obtaining a perfect score for Recall.

TABLE II
PERFORMANCE COMPARISON OF THE PROPOSED APPROACH WITH BSS, ADAPTIVE FILTERING, AND DENOISING APPROACHES USING REAL DATASET 2 IN TERMS OF STATISTICAL MEASURES OF TP, FP, FN, PRECISION, RECALL, AND F1-SCORE.

	TP	FP	FN	Precision	Recall	F1
Proposed Approach	9	1	0	0.90	1.00	0.95
BSS Approach	9	5	0	0.64	1.00	0.78
Adaptive Filtering	9	6	0	0.60	1.00	0.75
Denoising Approach	-	-	-	-	-	-

Further results using longer portions of this dataset are shown in Figure 21 and Figure 22 marked with TP, FP, FN events. These figures do not include comparison with other algorithms due to limited space and the fact that these algorithm have already been shown to under-perform in Figure 20. The values for Precision, Recall, and F1-scores for Figures 21 and 22 are summarized in Table III.

Simulation results with these two real datasets provide good basis for comparing the performance of proposed technique with other algorithms. While most algorithms performed almost perfectly for the real dataset 1, their performances were much poorer for the real dataset 2 with the BSS algorithm

TABLE III
PERFORMANCE ANALYSIS OF THE PROPOSED APPROACH IN TERMS OF STATISTICAL MEASURES OF TP, FP, FN, PRECISION, RECALL, AND F1-SCORE FOR THE SIGNALS RECOVERED IN FIGURES 21 AND 22.

	TP	FP	FN	Precision	Recall	F1
Figure 21	23	3	3	0.88	0.88	0.88
Figure 22	25	4	2	0.86	0.93	0.89

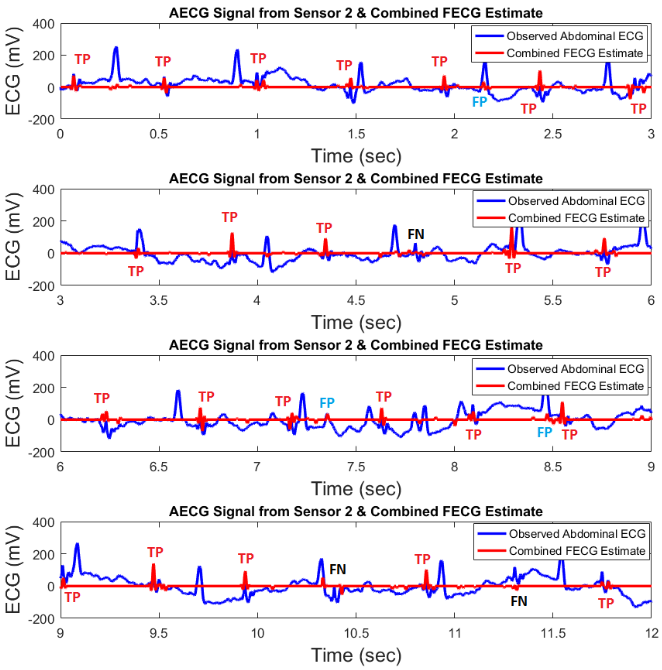

Fig. 21. Fetal ECG signal extracted from a 12 second portion of the recording from real dataset 2 marked with TP, FP, and FN events (Blue: Abdominal ECG, Red: Estimated fetal ECG).

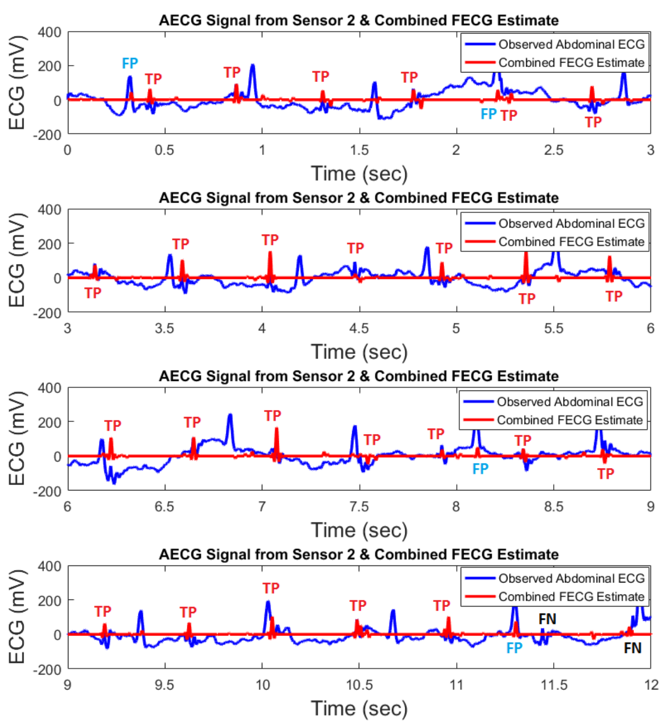

Fig. 22. Fetal ECG signal extracted from another 12 second portion of the recording from real dataset 2 marked with TP, FP, and FN events (Blue: Abdominal ECG, Red: Estimated fetal ECG).

failing completely in separating the MECG and FECG signals. The results from the proposed technique are however quite consistent and provide a significantly accurate extraction of the fetal ECG signal in both cases. These results demonstrate the potential of the proposed technique in providing useful information on the fetal cardiac activity while dealing with challenging ECG acquisition conditions and datasets.

V. Discussion and Conclusion

The availability of the ECG data from multiple electrodes provides a valuable opportunity to extract fetal ECG signals by taking advantage of the collective information in a collaborative formulation. We presented a comprehensive new scheme for the joint sparse estimation and extraction of fetal ECG signals from the abdominal ECG recordings of pregnant women. The proposed scheme exploited the support similarities between the ECG signals when they are represented in standard wavelet and specifically learned dictionary domains. We also proposed technique of combining the estimates from different abdominal ECG electrodes based on the quality of the estimates from the individual sensors, enabling inference of a single fetal ECG source signal. Simulation results obtained using real datasets demonstrate the effectiveness of the proposed scheme in eliminating the mother's ECG signal component from the abdominal ECG recordings, and in the removal of the noise and distortions from the recovered fetal ECG estimates. It is clearly shown that the proposed joint sparse estimation approach provides an efficient framework to extract the fetal ECG source signal with good estimation quality and accuracy compared with other state-of-the-art algorithms, specially when dealing with challenging ECG data. The proposed technique is able to demonstrate good performances with even a few seconds duration of signals available from the chest of the pregnant woman to facilitate the MECG dictionary learning process. We also note that the proposed technique works requires a minimum of just two ECG signals available from the abdomen for the proposed MMV based approach to work effectively. These conditions can be met quite easily as ECG data is generally available from more than one electrode at both the chest and the abdomen. The proposed scheme can prove to be a valuable tool in the monitoring of the fetal cardiac activity, enhancing access to critical insights while utilizing standard non-invasive ECG acquisition methodologies.

VI. Acknowledgement

This work was funded by King Abdullah University of Science and Technology (KAUST), Thuwal, Saudi Arabia.

References

[1] F Marzbanrad, Y Kimura, K Funamoto, S Oshio, M Endo, N Sato, M Palaniswami, and A H Khandoker, "Model-Based Estimation of Aortic and Mitral Valves Opening and Closing Timings in Developing Human Fetuses," *IEEE Journal of Biomedical and Health Informatics*, vol. 20, no. 1, pp. 240–248, jan 2016.

[2] M J Rooijakkers, C Rabotti, H de Lau, S G Oei, J W M Bergmans, and M Mischi, "Feasibility Study of a New Method for Low-Complexity Fetal Movement Detection From Abdominal ECG Recordings," *IEEE Journal of Biomedical and Health Informatics*, vol. 20, no. 5, pp. 1361–1368, sep 2016.

[3] Andrew T Reisner, Gari D Clifford, and Roger G Mark, "The physiological basis of the electrocardiogram," *Advanced methods and tools for ECG data analysis*, pp. 1–25, 2007.

[4] D Castro, P Flix, and J Presedo, “A Method for Context-Based Adaptive QRS Clustering in Real Time,” *IEEE Journal of Biomedical and Health Informatics*, vol. 19, no. 5, pp. 1660–1671, sep 2015.

[5] Michiyoshi Sato, Yoshitaka Kimura, Shinichi Chida, Takuya Ito, Norihiro Katayama, Kunihiro Okamura, and Mitsuyuki Nakao, “A novel extraction method of fetal electrocardiogram from the composite abdominal signal,” *Biomedical Engineering, IEEE Transactions on*, vol. 54, no. 1, pp. 49–58, 2007.

[6] José Luis Camargo-Olivares, Rubén Martín-Clemente, Susana Hornillo-Mellado, M M Elena, and Isabel Román, “The maternal abdominal ECG as input to MICA in the fetal ECG extraction problem,” *Signal Processing Letters, IEEE*, vol. 18, no. 3, pp. 161–164, 2011.

[7] Maria G Jafari and Jonathon A Chambers, “Fetal electrocardiogram extraction by sequential source separation in the wavelet domain,” *Biomedical Engineering, IEEE Transactions on*, vol. 52, no. 3, pp. 390–400, 2005.

[8] Ashish C Bhatia and Robin L Travers, “extraction of fetal contribution to ECG recordings using cyclostationary-based source seperation method,” *Seminars in cutaneous medicine and surgery*, vol. 31, no. 3, pp. 151–2, 2012.

[9] Shao Wenting Shao Wenting, Fang Bin Fang Bin, Wang Pu Wang Pu, and Ren Mingrong Ren Mingrong, “FECG Extraction Based On BSS Of Sparse Signal,” *2nd International Conference on Bioinformatics and Biomedical Engineering*, , no. 52002011200706, pp. 1457–1460, 2008.

[10] V. Zarzoso and a. K. Nandi, “Noninvasive fetal electrocardiogram extraction: Blind separation versus adaptive noise cancellation,” *IEEE Transactions on Biomedical Engineering*, vol. 48, no. 1, pp. 12–18, 2001.

[11] V. ZARZOSO, A. K. NANDI, and E. BACHARAKIS, “Maternal and foetal ecg separation using blind source separation methods,” *Mathematical Medicine and Biology: A Journal of the IMA*, vol. 14, no. 3, pp. 207–225, Sept 1997.

[12] Zhilin Zhang, Tzyy-Ping Jung, Scott Makeig, and Bhaskar D Rao, “Compressed sensing for energy-efficient wireless telemonitoring of noninvasive fetal ECG via block sparse Bayesian learning.,” *IEEE transactions on bio-medical engineering*, vol. 60, no. 2, pp. 300–309, 2013.

[13] Reza Sameni, Christian Jutten, and Mohammad B. Shamsollahi, “What ICA Provides for ECG Processing: Application to Noninvasive Fetal ECG Extraction,” *2006 IEEE International Symposium on Signal Processing and Information Technology*, vol. c, pp. 656–661, 2006.

[14] Vincent Vigneron, Anisoara Paraschiv-Ionescu, Annabelle Azancot, Olivier Sibony, and Christian Jutten, “Fetal electrocardiogram extraction based on non-stationary ICA and wavelet denoising,” in *Signal Processing and Its Applications, 2003. Proceedings. Seventh International Symposium on*. IEEE, 2003, vol. 2, pp. 69–72.

[15] L. de Lathauwer, B. de Moor, and J. Vandewalle, “Fetal electrocardiogram extraction by blind source subspace separation,” *IEEE Transactions on Biomedical Engineering*, vol. 47, no. 5, pp. 567–572, May 2000.

[16] Maha Shadaydeh, Yegui Xiao, and Rabab Kriedieh Ward, “Extraction of fetal ECG using adaptive Volterra filters,” *16th European Signal Processing Conference (EUSIPCO 2008)*, , no. Eusipco, pp. 1–5, 2008.

[17] Joachim Behar, Alistair Johnson, Gari D Clifford, and Julien Oster, “A comparison of single channel fetal ECG extraction methods,” *Annals of biomedical engineering*, vol. 42, no. 6, pp. 1340–1353, 2014.

[18] O.D. Escoda and L. Granai, “Ventricular and atrial activity estimation through sparse ECG signal decompositions,” *Acoustics Speech and Signal Processing, 2006 IEEE International Conference on*, vol. 2, pp. 1060–1063, 2006.

[19] Iulian B Ciocoiu, “Single channel fetal ECG recovery using sparse redundant representations,” in *Signals, Circuits and Systems (ISSCS), 2011 10th International Symposium on*. IEEE, 2011, pp. 1–4.

[20] Shuicai Wu, Yanni Shen, Zhuhuang Zhou, Lan Lin, Yanjun Zeng, and Xiaofeng Gao, “Research of fetal ECG extraction using wavelet analysis and adaptive filtering,” *Computers in biology and medicine*, vol. 43, no. 10, pp. 1622–1627, 2013.

[21] V. Vigneron, a. Paraschiv-Ionescu, a. Azancot, O. Sibony, and C. Jutten, “Fetal electrocardiogram extraction based on non-stationary ICA and wavelet denoising,” *Proceedings - 7th International Symposium on Signal Processing and Its Applications, ISSPA 2003*, vol. 2, pp. 69–72, 2003.

[22] Paul S Addison, “Wavelet transforms and the ECG: a review,” *Physiological Measurement*, vol. 26, no. 5, pp. R155–R199, 2005.

[23] A. Khamene and S. Negahdaripour, “A new method for the extraction of fetal ecg from the composite abdominal signal,” *IEEE Transactions on Biomedical Engineering*, vol. 47, no. 4, pp. 507–516, April 2000.

[24] M. G. Jafari and J. A. Chambers, “Fetal electrocardiogram extraction by sequential source separation in the wavelet domain,” *IEEE Transactions on Biomedical Engineering*, vol. 52, no. 3, pp. 390–400, March 2005.

[25] Christian andVerleysen Michel Vrins, Frédéric andJutten, “Sensor array and electrode selection for non-invasive fetal electrocardiogram extraction by independent component analysis,” in *Independent Component Analysis and Blind Signal Separation*, Carlos G. Puntonet and Alberto Prieto, Eds., Berlin, Heidelberg, 2004, pp. 1017–1024, Springer Berlin Heidelberg.

[26] P. P. Kanjilal, S. Palit, and G. Saha, “Fetal ecg extraction from single-channel maternal ecg using singular value decomposition,” *IEEE Transactions on Biomedical Engineering*, vol. 44, no. 1, pp. 51–59, Jan 1997.

[27] G. Camps, M. Martinez, and E. Soria, “Fetal ecg extraction using an fir neural network,” in *Computers in Cardiology 2001. Vol.28 (Cat. No.01CH37287)*, 2001, pp. 249–252.

[28] Q. Yu, Q. Guan, P. Li, T.-B. Liu, J.-F. Si, Y. Zhao, and Y.-Q. Liu, H.-X.and Wang, “Wavelet optimization for applying continuous wavelet transform to maternal electrocardiogram component enhancing,” *Chinese Physics B*, vol. 26, no. 11, pp. 118702, Oct. 2017.

[29] Mohammad Niknazar, Bertrand Rivet, and Christian Jutten, “Fetal ECG extraction by extended state Kalman filtering based on single-channel recordings,” *Biomedical Engineering, IEEE Transactions on*, vol. 60, no. 5, pp. 1345–1352, 2013.

[30] Dragos-Daniel Taralunga, G. MihaelaUngureanu, IlincaGussi, RodicaStrungaru, and Werner Wolf, “Fetal ecg extraction from abdominal signals: A review on suppression of fundamental power line interference component and its harmonics,” in *Comp. Math. Methods in Medicine*, 2014.

[31] Mudassir Masood and Tareq Y Al-Naffouri, “Support agnostic Bayesian recovery of jointly sparse signals,” in *Signal Processing Conference (EUSIPCO), 2014 Proceedings of the 22nd European*. IEEE, 2014, pp. 1741–1745.

[32] M. Aharon, M. Elad, and A. Bruckstein, “ K-SVD: An Algorithm for Designing Overcomplete Dictionaries for Sparse Representation,” *IEEE Transactions on Signal Processing*, vol. 54, no. 11, pp. 4311–4322, 2006.

[33] Lieven De Lathauwer, Bart De Moor, and Joos Vandewalle, “Fetal electrocardiogram extraction by source subspace separation,” in *Proc. IEEE SP/ATHOS Workshop on HOS, Girona, Spain*, 1995, pp. 134–138.

[34] E B Mazomenos, D Biswas, A Acharyya, T Chen, K Maharatna, J Rosengarten, J Morgan, and N Curzen, “A Low-Complexity ECG Feature Extraction Algorithm for Mobile Healthcare Applications,” *IEEE Journal of Biomedical and Health Informatics*, vol. 17, no. 2, pp. 459–469, mar 2013.

[35] Glass L Hausdorff JM Ivanov PCh Mark RG Mietus JE Moody GB Peng C-K Stanley HE Goldberger AL, Amaral LAN, “PhysioBank, PhysioToolkit, and PhysioNet: Components of a New Research Resource for Complex Physiologic Signals,” *Circulation*, vol. 23, no. 101, pp. e215–e220, jun 2000.